Game of Life: simple interactions ecology

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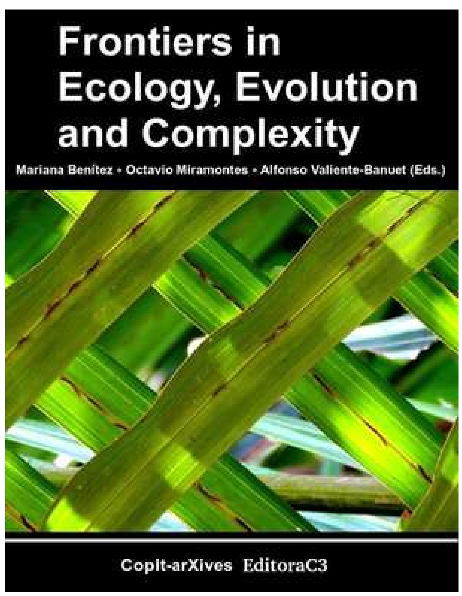
This is a separate chapter from the open access book

Frontiers in Ecology, Evolution and Complexity

Mariana Benítez, Octavio Miramontes & Alfonso Valiente-Banuet (Editors)
CopIt-arXives, 2014
Mexico City
ISBN: 978-1-938128-05-9



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1 Abstract

Alan Turing in his work of 1952 stated the importance of the analysis of pattern formation in biological systems focusing in color patterns and subject to chemical mechanisms, but his work was limited because of the mathematical complications of the model. However he had the clarity to propose the need of a new paradigm to face such biological problems. It was until the late seventies that John Conway proposed a mathematical game, based on cellular automata, with a clear biological similarity, where the most basic entities, called cells, can be in one of two possible states, conceptualized as dead or alive. The proposed rules give the game a critical trait, this is, within a specific threshold the cells can stay alive otherwise they will become dead. This game is known as Conway's Game of Life. The emergence of new patterns is one of the properties of Conway's Game of Life, another is the great diversity of forms that it can span. All of this depend on the initial conditions of the game. Along this chapter we propose Conway's Game of Life as an alternative model to study ecological systems.

2 Resumen

Alan Turing en su trabajo de 1952 planteó la importancia del análisis de la formación de patrones en los sistemas biológicos enfocándose en los patrones de color y restringiéndose a mecanismos principalmente químicos, pero su trabajo quedó acotado por la complicación derivada del modelo matemático propuesto. Sin embargo, tuvo la claridad de establecer la necesidad de plantear un nuevo paradigma para abordar este tipo de problemas biológicos. Fue hasta los setentas que John Conway propuso un juego matemático, basado en autómatas celulares, con un claro símil biológico, donde las entidades más básicas son celdas con dos estados posibles, conceptualizadas como vivas o muertas. Las reglas propuestas le dan un un caracter crítico al juego, esto es, dentro de un umbral es-

pecífico las células pueden seguir viviendo y fuera de este umbral la supervivencia no es posible. A este juego se le conoce como el Juego de la Vida de Conway. La emergencia de patrones no preestablecidos es una de las propiedades de este juego, otra característica importante es la gran variedad de formas que pueden generarse. Todas estas dependientes de las condiciones iniciales con las que comience la evolución del Juego de la Vida. A lo largo del capítulo proponemos al Juego de la Vida como una alternativa de modelo para estudiar a los sistemas ecológicos.

3 Introduction

Since the publication of the article *The chemical basis of morphogenesis* written by Alan Mathinson Turing in 1952, the study of morphogenetic patterns has taken a surprising turn. Turing proposed a model to explain how patterns emerge during development. The purpose of his work was to discuss a possible mechanism by which the genes of a zygote can determine the anatomical structure of the resulting body. To do this he proposed that by using certain known physical laws it is possible to explain many of the phenomena of morphogenesis. His proposal suggests that a system of chemicals called morphogens (for qualities in the generation of shapes) react together and diffuse through the tissue, which can help to account for the main phenomena of morphogenesis. Such a system, although originally it can start from a quite homogeneous state, after a while may develop a pattern or structure due to instability of homogeneous equilibrium, which is triggered by some random disturbances [1, 2].

This model describes the evolution of the system in two parts: the mechanical and the chemical. The mechanical part of the system describes positions, masses, speeds and elastic properties of the cells, and also the forces interacting among them. The continuous form of the model generates essentially the same information but in the form of tension, speed, density and the elasticity of the material.

The chemical part of the system is given by the chemical composition of an individual cell and the rate of diffusion of each substance between two adjacent cells. In the continuous form of the model, the concentrations and diffusion rates of each substance must be determined for each point in the system and for every moment.

Although Turing did not consider all of the aspects proposed in his reaction-diffusion model (hereafter RD), he postulated that in order to compute the system's evolution one must take into account changes in the position and changes and velocity which are given by Newton's laws of motion. He also considered osmotic pressure, tensions generated by the system's elasticity and movement as given by chemical information, dissemination of chemicals. Regions where such diffusion is possible are given by mechanical information.

Having in mind some of these postulates Turing presented a mathematical model based on nonlinear partial differential equations with constant coefficients. In it he established the basis for the development of models, now known as reaction-diffusion (RD) models (see review in [2–5]).

Since the development of Turing's RD model, the theoretical study of the emergence of patterns in morphogenesis has received a lot of attention including research on color patterns studied by dynamic generic RD models [1, 3]. But oftne RD models do not consider the cellular and tissue environment in which these patterns emerge (see [6–9]), it is therefore important and useful to integrate in the same model the mechanical and chemical aspects considered in the Turing model, aiming at an explanation supported by experimental biological evidence, about the mechanisms involved in the emergence and maintenance of patterns in morphogenesis.

One of the main interests driving Turing in his work was the need for a deep understanding of the formation of patterns, but as he himself states it, when many different properties are included in this problem, such as electrical and mechanical properties, not only the chemical aspects originally considered, the complexity of the problem escalates rapidly. Thus he pointed out the need to develop a paradigm where the exploration of different environments, and conditions over these environments, could be stated in a more natural and biological fashion, setting the basis for further exploration and analysis on a very complex and widely misunderstood topic.

Following this reasoning, we propose to describe a framework where we can include aspects like tissue and physical mechanisms involved in the study of cellular interactions during morphogenesis which were put aside in Turing's original proposal, although they were mentioned as important for understanding the general problem of morphogenesis. In contrast with classical mathematical techniques the framework that we are seeking needs to be able map the different properties of the living systems in a more natural way.

In the 1970's, John Conway described a mathematical game named The Game of Life or Conway's Life that makes an analogy with living cells. That is, if a living cell is overcrowded or extremely isolated it will die, and it will survive only if it has the right number of neighbors around it. This game has many interesting properties and it will be described in the next section.

Given that Ecology is the science that studies the interactions between sets of biotic and abiotic factors we can establish an analogy with Conway's Life through the Ecology that emerges through a basic and simple set of rules that make a diverse zoology of patterns (see [10]) which coexist and interact giving rise to a set of complex interactions in a self-organized way. This contrasts with the general perception that every system requires a different history and that a completely different set of rules is needed to describe such a system. It is very important to specify that the simple rules we are talking about are not the ones that give rise to different patterns in the skins of animals or at least not only that set of rules (which can be interesting in itself) but include those rules that apply to any kind of system that is known to show emergent properties through the interaction of autonomous entities within it.

Historically biology has understood biological mechanisms as objects that are restricted by matter in a physical and tangible way. Modeling in biology is usually carried out in a canonical fashion incorporating all of the interacting aspects of the modeled phenomenon in an ad-hoc way, but it is often forgotten that restrictions interacting with the systems are an intrinsic part of them. This meaning that such restrictions are not just a different dimension added on to a biological one, but are an active part of this biological dimension.

In contrast with this historical vision of how modeling is carried out in biology, the proposed alternative framework seeks to rethink this modeling process by starting with a set of simple and representative rules for all of the different entities interacting in the phenomenon selected to be modeled and letting the devised rules make the system evolve long enough for all of the entities that cannot coexist gets eliminated through such evolution.

As we will see in the following sections, the Game of Life provides an excellent example of how a set of simple rules can span a diverse zoology, not only designing the final form that each member of this zoology should have but also defining the interactions among them, therefore providing a framework where we can really talk, experiment with and analyze an ecology of biological patterns.

4 Computation, Cellular Automata and Conway's Game of Life

One of the most interesting and profound debates in Biology concerns form and function. Which one determines the other? Throughout time from ancient Greek philosophers to the present day scientists, such as biologists have given much attention to this specific question.

This question has received a lot of attention from a great and diverse community of scientists but at this point we are going to make reference to two who which have made landmark contributions in the field of Mathematics and Computer Science.

The first is Alan M. Turing, part of whose work was described in the first part of the present chapter. The second is the Hungarian John von Neumann, a prolific mathematician, who inspired by the work of Alan Turing, worked in the 1940s on the technical and philosophical problem of self replication. His main aim was to determine the specific set of rules by which a machine could replicate itself. He devised an automaton D structured in the following way: One part is the functional structure A. A second part is the structure that copies the set of instructions I_A while a third part C inserts the copy of the instructions into the new structure [11]. This is what was later known as a cellular automaton.

In the late 60s John Conway worked on a form derived from than in Neumann's work. He and his students devised an automaton with a simpler set of rules than Neumann's but with some important mathematical inclusions, such as the capacity universal computation [12]. He then came up with a recreational mathematical construction named by Conway himself as *Life*, or as the Game of Life as we know it, which was published in [13].

As said in the previous paragraph one of the most important characteristics of Conway's Game of Life is its capacity to perform universal computation, meaning that what every other computer can do, can be done with this specific cellular automaton, given the proper initial conditions.

The usual concept of computation sees it as a synonym of calculation. This is not wrong only incomplete. The concept of computation in cellular automata (CA) can be understood in the following two ways [14]:

- If the CA is understood as the program, then the mapping between the initial configuration (input) and the final state (output) is what we can call computation.
- A more theoretical approach would be to select a very specific input that can emulate any other computer (universal computation). However this interpretation is extremely difficult to go into in depth and its valuable results are restricted to a computational sciences theoretical frame.

The biological and physical importance of the concept of computation resides in the relationship between discrete and dynamical systems, and the utility of different models designed to understand natural phenomena. Bearing this in mind we now can describe the general aspects of Conway's Game of Life as follows. A group of cellular automata will be arranged in a bidimensional lattice in which every cell can be in two states, 0 to resemble the *dead* state and 1 to resemble *alive* state. The value of every cell will be determined by the state of its neighbors by the following rules:

- Every cell remains dead if its surroundings are dead. Meaning that if no more than 2 two cells are alive, the present cell will remain dead.
- Every cell will die from overcrowding or solitude. Meaning that a cell which is alive will die if it has more than three living or less than two living cells adjacent.
- Every cell will come to life if it has three living neighbors.
- Every new cell stays alive if it has either two or three living neighbors. This establishes that life can exist in a small and fair well determined range of eight possible values.

All this rules are updated in a synchronous way throughout the whole lattice [15, 16].

A great diversity of different patterns emerged from this small set of rules, which can be classified into three main groups: a set of static objects, a set of objects with periodic forms but static in space, and a set of moving objects. Some examples can be seen in figures 1, 2, 3 and 4.

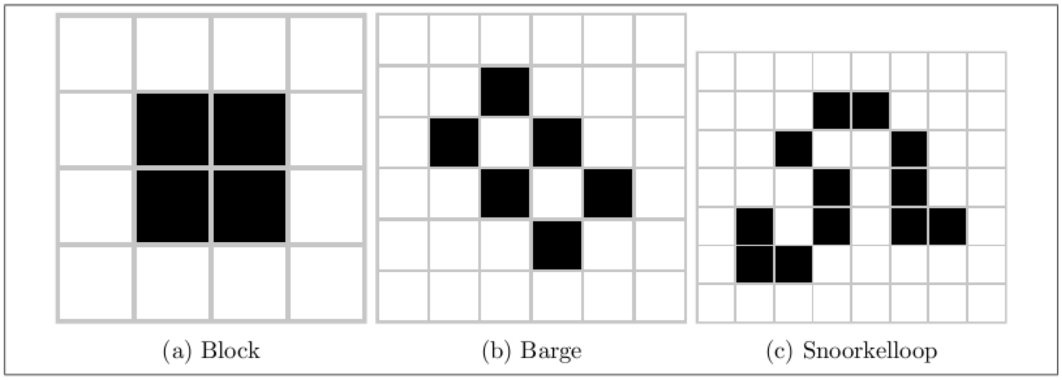

Figure 1: Three still forms of life which can serve as memory.

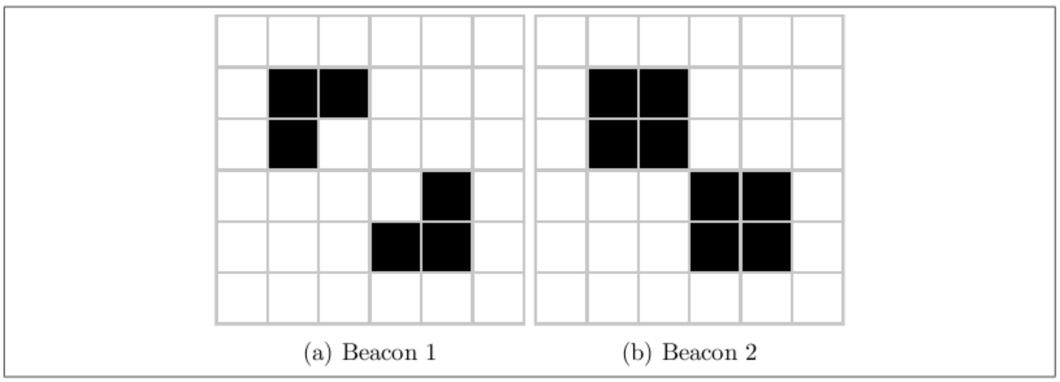

Figure 2: Both steps of a period-2 oscillator named Beacon.

The importance of these types of objects formed in Conway's Game of Life comes from the fact that to carry out universal computation the combination of the former elements is needed in the following way.

In order to compute anything it is easy to realize that the ability to count and to store information is required. This is accomplished by the periodic and static classes respectively. That is, the still elements work as memory elements and the periodic elements can aid as counting entities. The third class, namely the class of moving objects class, carries out the task of transporting information. This particular aspect is of great importance, since our main interest is in the mechanism to communicate information in different scales.

These three types of elements are used to prove that Conway's Game of Life can perform universal computation. A sketch of universality of computation can be seen in [16].

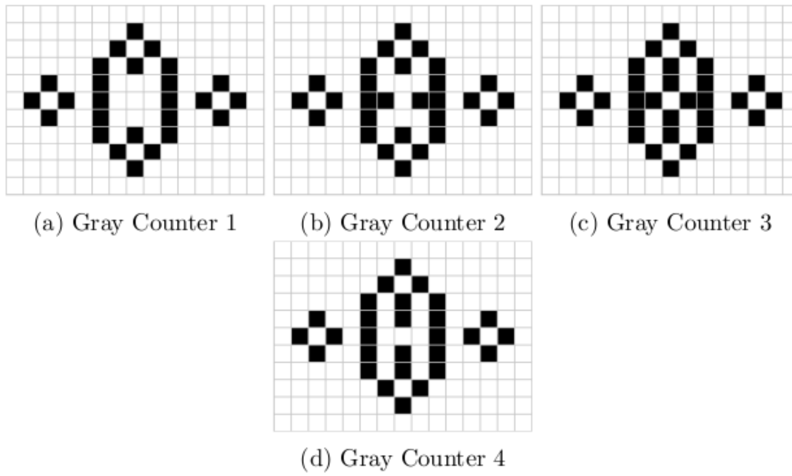

Figure 3: A period-4 oscillator named Gray Counter.

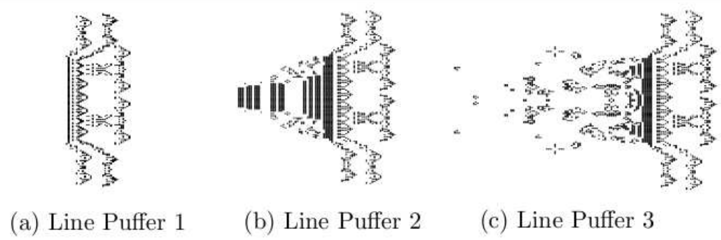

Figure 4: A puffer that acts like a *train*, except only that it leaves debris behind

5 Simple Rules and Pattern Formation

The Game of Life is a particular case of cellular automaton but the cellular automata model is widely used to model patterns using simple interactions. For an extensive review you can check [9].

During the 1980s and early 1990s Stephen Wolfram did an extensive research into a simplified form of a cellular automaton, one that can evolve in only one spatial dimension and one temporal dimension. This automaton consists only two possible states to describe and a radius. This is the so called Elemental Cellular Automata which is an automata whose actual state can be affected by its present state and the one of its nearest neighbors. That is:

$$\sigma_n^{t+1} = f(\sigma_{n-1}^t, \sigma_n^t, \sigma_{n+1}^t) \tag{1}$$

Wolfram's work is an extensive qualitative and quantitative exploration of all the possible elemental cellular automata that can be formed with this elemental configuration. From this research he came up with a widely used classification of all possible cellular automata formed with this configuration and based upon statistical properties of different configurations. His work can be read in [17] and with more technical detail in [18]. The classification of the different types of CA is shown in the following table.

Class	Description
1	Evolution rapidly leads to a unique homogeneous state, that is they evolve to a fixed point pattern.
2	Evolution, after some iterations that depends on some appropriate initial sequence, leads to a set of values that are either stable or periodic.
3	Almost all initial states lead to chaotic patterns with such statistical properties that makes it difficult to extract structure out of them.
4	The most interesting class of them all since even when they seem to reach a quiescent state, some of them show persisting spatial structures that repeat in time, that is, structures that can propagate indefinitely through the evolution of the cellular automata. The Game of Life is known to be in this class, also the Elementary Cellular Automata rule 110.

Table 1: Classification of different rules derived from Wolfram's work

Our main interest in the Game of Life is precisely the fact that there is speculation that it is a Class 4 CA and that out of very simple and biological-like rules a varied and rich range of patterns can be formed. Bak et al showed in [19] that this game can be in a critical regime, meaning that structures of all scales are present at one given time. Also, as stated before, the Game of Life is a self-organized system, meaning that it is driven by evolution of the system. Thus, as shown by Per Bak et al., this simple set of rules can produce a self-organized criticality.

The importance of this fact is that when the system is in a critical state it can provide a mechanism for the emergence of scale-free structures [19] and therefore resembles at least in a way the mechanisms used by Nature to form all sorts of structures and patterns. Some examples of recent findings can be seen in [20, 21].

6 Conclusions

As we have mentioned, Conway's Game of Lfe is a versatile mathematical and computational tool with some interesting particularities. The first is the limited and simple set of rules describing it. The second is the fact that it is a zero player game, that is, it is a game driven by the evolution of the initial condition, a set of initial living and dead cells, with successive iterations of the simple set of rules previously described. The third is the apparently abundant zoology that can be produced with them. It is important to note that the Game of Life is reported to be, not without debate [14, 22], to be a class 4 cellular automaton, suggesting that the ability to create all of this ecology of living creatures (in a broad sense of "living" and "creatures") is partly related to the fact that a set of rules operating in a critical regime is necessary in order to create this diverse set of patterns.

Another interesting aspect that needs to be remarked is the fact that in this computational game, unlike many canonical mathematical structures, form is function, meaning that specific forms induce specific functions. For example, if we alter just one neighbor in some of the objects presented like the oscillators (figure 2) or the puffer (figure 4) we may completely alter its function. This is observable even in the static forms in the Game of Life like the still objects (figure 1). All these aspects taken in consideration imply that synchronization and specific structural issues need to be taken into account in order to present a functional Game of Life that can carry universal computation.

This small computational self-organized game is a good start to model and comprehend the importance of the rules and the interactions between the forms they generated through them in order to rise produce great ecological diversity. It is an alternative way to understand such a system instead of the usual approach of constructing different histories and sets of working rules for every system. In order to create new and possibly more complex forms the rules can be used in a recursive fashion with multiple iterations. It is important to note that if the chosen rules work well with the formation of small scale structures then there is no apparent need to choose some other rules until they stop working. All the forms created under this framework can be tested together, forming bigger ensembles at each time step, and the persistence in time of such ensembles will depend entirely of their surroundings.

In this context it is important to note that in the biological world we are surrounded by "historical contingencies" wherever we look. These contingencies include the evolution of genetic information coding phenotypical characteristics and the fact that such genetic information is subject to changes in the environment. Another example is the way biological diversity is reshaped by the different mass extinctions registered in biological history. All these examples can be modeled through a CA with a layered model in which the upper layer interacts with the bottom layer in a stochastic fashion. But the central point about the effects of historical contingencies is that evolution in biology is not determined by the initial conditions of the system. This is not the case in the Game of Life, which functions by a deterministic set of rules, since every final state is determined by a specific initial condition which the game starts with. We believe that further research needs to done on this topic.

Another topic for further research is the fact that the structure of biological organisms is modular and hierarchical. Could it be possible that dynamics determined by a Conway's Game of Life set of rules can show this specific aspect?

If we take Conway's Game of Life as our paradigm, then we can say that patterns emerge under diverse circumstances subject to ecological interactions. These seems as very sophisticated patterns however it also seems that a great diversity of patterns can emerge under the restrictions of simple rules: that is, that the interactions among different biological factors can span a great number of different form of life but we can only account for just a small group of conserved patterns.

And extending this particular subject the reader could ask whether only celullar automata defined in Conway's Game of Life are capable of spanning such a diverse interacting zoology with such properties. Could another type of automaton give rise to such a rich environment? So, a further research topic can be precisely this: what is the simplest automaton capable of spanning this diverse ecosystem? The reasons why we chose the Game of Life is because of its natural and biology-like set of rules which we think makes it easier to understand and reinterpretate for this purpose.

As we have seen in the Game of Life we only have a small set of rules which that nevertheless can span complex patterns which maintain a close resemblance to biological life as we know it.

As we know, in biological life we can have basal states and modifications of these basal states generated throughout their evolution. So we can ask ourselves whether it is the diversity of the forms of life which must be accounted for as a possible outcome, even when life itself strike us as limited.

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