Randomness in Biology

by
Pedro Miramontes

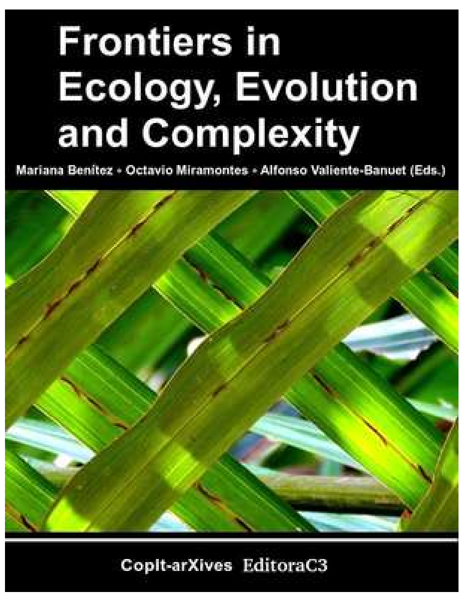
Departamento de Matemáticas, Facultad de Ciencias, UNAM, Mexico City
Centro de Ciencias de la Complejidad C3, UNAM, Mexico City

This is a separate chapter from the open access book

Frontiers in Ecology, Evolution and Complexity

Mariana Benítez, Octavio Miramontes & Alfonso Valiente-Banuet (Editors)
CopIt-arXives, 2014
Mexico City
ISBN: 978-1-938128-05-9

http://scifunam.fisica.unam.mx/mir/copit/TS0012EN/TS0012EN.html



Contents

Randomness in Biology

P. Miramontes, Facultad de Ciencias UNAM, Mexico

1 Abstract

Using concepts of dynamical systems theory a formal framework of randomness is advanced. Given the fact that chance plays an important role in biology and specifically in evolutionary biology, the purpose of this essay is to reevaluate, under this framework, the way we perceive how biological phenomena may be operating.

2 Resumen

El azar juega un papel muy relevante en la biología. Aquí se propone un nuevo enfoque para lo que se debe de entender como aleatorio y se espera que dicha propuesta modifique la manera como percibimos los fenómenos biológicos.

3 Introduction

The concept of randomness is deeply ingrained in biology. Specifically in evolutionary biology where it plays a major role in the Neodarwinian scheme where the major, or even unique, source of genoptypic variation is random mutations. As a matter of fact, in the 1940s and 1950s, the Modern Synthesis fused the Darwin's principle of natural selection with Mendelian inheritance to create the Synthesis where the gradual and random allelic substitution is the only source of evolutionary change [1]

We now know that this is far from being true because there are some mechanisms contributing to the genotypic diversity that are not either random or small, in the Darwinian sense. Just for example: gene and genome duplication, horizontal gene transfer or symbiogenesis [2].

As of now, it seems to be that there is a vast majority of scholars accepting that the source of biological variation are random but its products are not and that the origins of biological order is natural selection. Then the lack of a clear definition of what should be understood by random is somewhat shocking. Most of the people feel comfortable with the dictionary definition of randomness as "Having no definite aim or purpose; not sent or guided in a particular direction; made, done, occurring, etc." Phrased this way (Oxford Dictionary) it seems tailored in a circular way to fulfill the Neodarwinian needs.

4 Chaos and Time Series

Irregular, uncorrelated behavior in biology used to be conceived as random without any further inquiry and with no analysis of the meaning of randomness. Most of the times it is modeled by adding white noise to a deterministic signal. The sharp separation between determinism and randomness was originated by a wrong identification of determinism with predictability. This scheme is being abandoned as our knowledge of chaos is spreading. The possibility of finding unpredictable behavior in a deterministic system was a shock for the advocates of the dichotomy determinism–randomness. Thanks to the seminal papers of Edward Lorenz [3] and Robert May [4] [1] chaotic systems have today a legitimate place in science.

Chaotic behavior is deterministic and unpredictable at the same time and notwithstanding it has a number of hidden regularities that allow the researcher to measure the degree of correlation (or lack of), the predictability horizon and the structure of the long-term behavior, the so-called *Attractor*. Before the advent of chaos theory, an attractor could be a point attractor or a periodic cycle. At present, it is well known that chaotic regimes give birth to attractors having fractal structure (the opposite is not always true). Ruelle and Takens christened them as *strange attractors*.

When collecting data from the field or the lab, a biologist usually gets it organized as a sequence of one or many variables taken at uniform time intervals. This is a *Time Series* and mathematically is expressed as a sequence

$$\{x_i\}_{i=1}^{n} \tag{1}$$

An actual time series from an experiment or census is hardly random [2]. Then it should be the outcome of a dynamical system, if this is the case, it could be an iterated system $x_{i+1} = f(x_i)$ or the discretization of a differential equation $x_{i+1} = hf(x_i) + x_i = g(x_i)$.

5 Attractor Reconstruction

A very important and yet unsolved, in general, task is to find the dynamical system that produces a given the time series. A celebrated advance in this direction is the embedding

[1] The chaotic behavior was already seen by Henri Poincaré by the end of the XIX century. See [5]

[2] In any sense of the term. Accepting the opposite would mean that there are no natural laws

theorem by F. Takens [6]. In 1980 he demonstrated that to find the dimension (the number of state variables) of a dynamical systems it is enough to know the output of one of them to reconstruct the attractor.

Let us assume that we have a scalar (one dependent variable) time series and that the underlying dynamics has an strange attractor embedded in an space whose dimension is yet to be determined (point or periodic attractors are easily detected by many ways so they are excluded from now on). The procedure is as follows:

- Construct a vector time series from the original one by pairing consecutive values allowing a time gap

$$x_i \mapsto (x_i, x_{i+\tau}) \tag{2}$$

The τ parameter is chosen by trial and error. If τ is too small the points will be too close to each other and the main traits of the reconstructed attractor will be hidden in a flatten figure over the identity line in $\mathbb{R}^2$. If, on the other hand, τ is large the points will be uncorrelated and the plot will be a shapeless cloud. Now, repeat the procedure increasing the embedding dimension:

$$x_i \mapsto (x_i, x_{i+\tau}, x_{i+2\tau}, \ldots, x_{i+n\tau}) \tag{3}$$

until the attractor geometrically fully develops.

Figure 1 shows the Lorenz Attractor. It is the destination set of a system of three differential equations. The left figure is the phase space of the solution trajectories of the numerically integrated systems. On the right, the reconstructed attractor using only the data of one state variable following the procedure above outlined with $\tau = 8$. The issue is that having just a one-variable time-series it is not clear to know in advance how many degrees of freedom has the still unknown system. To overcome this problem the following procedure is recommended:

- Measure the fractal dimension of the vector set in $\mathbb{R}^2$. There is a number of methods to numerically estimate the fractal dimension of a set in $\mathbb{R}^n$. One of the faster and more accurate ones is the *correlation dimension* [7]:

$$C(\epsilon) = \lim_{\substack{n\to\infty \\ \epsilon\to 0}} \frac{g(\epsilon)}{N^2} \tag{4}$$

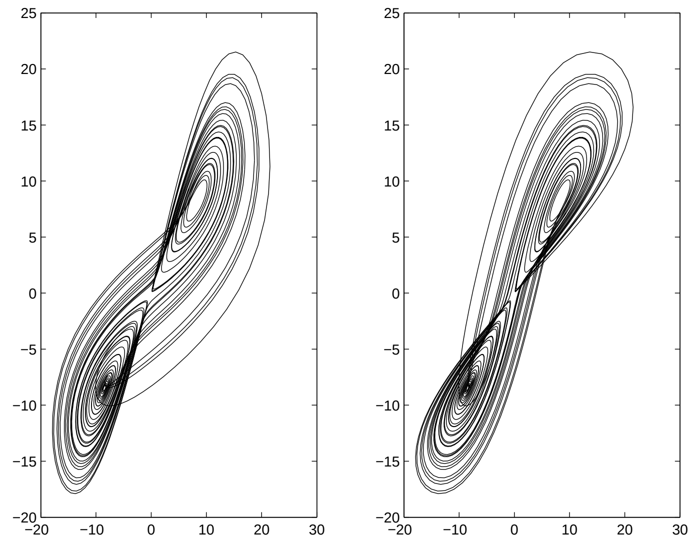


Figure 1: The Lorenz Attractor. The left figure Lorenz's System numerical solution. On the right, the attractor reconstruction after Takens' Theorem.

Where $g(\epsilon)$ stands for the number of points that are far from each other in less than ϵ. It is accepted that the correlation function scales as:

$$C(\epsilon) \sim \epsilon^d \tag{5}$$

where d is the *fractal (correlation) dimension*.

It is then straightforward to estimate the fractal dimension d as the slope of a line in a $\log(C(\epsilon))$ versus $\log(\epsilon)$. Once it is done, repeat the procedure and reevaluate the fractal dimension increasing the dimension of the embedding space. Plot the results in a diagram of the correlation dimension as function of the embedding dimension. The asymptote of the points (Figure 2) is the fractal dimension of the attractor and the integer number greater than it is the number of effective degrees of freedom of the putative dynamical system that produced the time series.

6 Discussion

Given a time series, it is then straightforward to follow the steps outlined above to reconstruct the putative attractor and to determine the number of state variables that are enough to engender it.

A white noise time series does not finds ever an asymptote; the straight line keeps going upwards as the embedding dimension growths. This is the signature of randomness. What we call chance, haphazard or stochastic is nothing more than high dimension

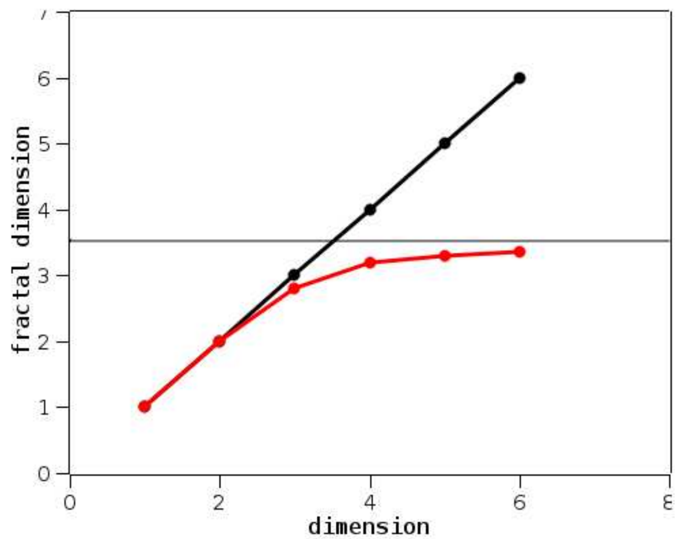


Figure 2: The asymptote of the red line is the dimension of the space that embedding the attractor. A white noise time series never bends down (black line)

chaos. It could be argued (mostly by mathematicians) that in any case randomness could be reached in the limit when the dimension approaches infinity and that those limits are never attained. This argument is easily treated when one reasons that the whole building of physics is founded in the model of the continuum of real numbers and that there seems to be no contradiction with the fact that physical objects are not continuous. Taking limits when one approaches zero or infinity should be thought as metaphors in Natural Sciences. So far I have shown that what we call randomness is nothing more that high–dimensional chaos[3].

Creationism often recur to the thought experiment of a typing monkey to claim the supposedly impossibility of Biological Evolution. Of course they are right of the monkey acts randomly. What they do not say is that evolution is restricted by Physical constraints that reduce the possible outcomes over which selections acts. Under constraints it is like the monkey typing over a doctored typing machine that does not allow forbidden dimers (in English the dimers "ww", "qq", etc. do not exist), trimers, tetramers, and having also limits over the word length. After all these restrictions the typing machine is not random anymore and it could be shown that it is colored noise and their reconstructed phase portraits are low dimensional. Nonetheless, there is no formal studies about the relationship of colored noise and chaos.

[3]An anonymous reviewer called my attention to the paper: "Chaos and Deterministic versus Stochastic Nonlinear Modeling" (Santa Fe Institute Working Paper, where Martin Casdagli advanced similar ideas years ago.)

There is fear to accept a deterministic Nature. This fear is understandable because "biological determinism" is a well characterized ideological posture that is frequently identified with political conservatism. There is nothing wrong with accepting a deterministic biology in the sense outlined in this essay even acknowledging that the ultimate components of biological systems should obey Quantum Mechanics where the notion of determinism gets blurred. The realm of Biology is far from the weird phenomena occurring in Quantum Mechanics; the Theory of Complex Systems teaches us that as we move up in the hierarchical ladder of the organization of matter, the laws governing the lower levels become irrelevant to describe the upper ones. To identify determinism with teleology is also a mistake since we understand the laws of chaos. Chaotical phenomena have no purpose and are unpredictable and, notwithstanding, are deterministic. Is there a real randomness? The kind of randomness illustrated by a Casino roulette?. The aim of this essay is to invite the community to view even this case as a case of ultimate determinism. In this framework, there is no point to treat as separate concepts randomness and high-dimensional chaos.

Randomness should be admitted as the mask we use to cover our lack of knowledge. Biology would gain a lot of understanding in the real meaning of its object of study if it accepts that behind what we call Randomness there are natural laws acting.

7 Bibliography

[1] S. Gould, "Is a New and General Theory of Evolution Emerging?" in *Self-Organizing Systems*, ser. Life Science Monographs, F. Yates, A. Garfinkel, D. Walter, and G. Yates, Eds. Springer US, 1987, p. 113–130.

[2] E. V. Koonin, "Darwinian evolution in the light of genomics," *Nucleic Acids Research*, vol. 37, no. 4, pp. 1011–1034, 2009.

[3] E. N. Lorenz, "Deterministic nonperiodic flow," *Journal of the Atmospheric Sciences*, vol. 20, no. 2, pp. 130–141, 1963.

[4] R. M. May, "Simple mathematical models with very complicated dynamics," *Nature*, vol. 261, no. 5560, pp. 459–467, 1976.

[5] F. Diacu and P. Holmes, *Celestial encounters: the origins of chaos and stability*. Princeton, NJ: Princeton Univ. Press, 1996.

[6] F. Takens, "Detecting Strange Attractors in Turbulence," in *Dynamical systems and turbulence*, D. A. Rand and L. S. Young, Eds. New York: Spinger-Verlag, 1980, p. 366–381.

[7] P. Grassberger and I. Procaccia, "Measuring the strangeness of strange attractors," *Physica D*, vol. 9, p. 189, 1983.